Clasa XI – M2 – prof. Cornelia Mestecan
Fişă de lucru – DERIVATE – FUNCŢII SIMPLE
Exerciţii rezolvate

Să se calculeze derivatele funcţiilor, scriindu-se de fiecare dată şi formulele utilizate:

a) $f:\mathbb{R}\to\mathbb{R},\ f(x)=3x^2-8x+2$; b) $f:\mathbb{R}\to\mathbb{R},\ f(x)=x^2+2^x$;

c) $f:\mathbb{R}\to\mathbb{R},\ f(x)=3e^x-5^x$; d) $f:\mathbb{R}\setminus\{0\}\to\mathbb{R},\ f(x)=5x^4+\frac{1}{x}+\frac{1}{x^3}$;

e) $f:\mathbb{R}\setminus\{0\}\to\mathbb{R},\ f(x)=\frac{x-4}{x}$; f) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{x-4}{x^2+2}$;

g) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\left(x^5-7x+2\right)e^x$; h) $f:(0;+\infty)\to\mathbb{R},\ f(x)=x\ln 2+\ln x$;

i) $f:(0;+\infty)\to\mathbb{R},\ f(x)=(x+3)\ln x$; j) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2\sin x+\cos x$;

k) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2x\sin x$; l) $f:(-1;1)\to\mathbb{R},\ f(x)=5\arcsin x-8\arccos x$;

m) $f:(-1;1)\to\mathbb{R},\ f(x)=\left(x^2+3\right)\arcsin x$; n) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{3\sin x-5}{\cos x+2}$;

o) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2arctgx+5arcctgx$; p) $f:(0;+\infty)\to\mathbb{R},\ f(x)=3\sqrt{x}+\sqrt[5]{x^3}\ \frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x^2}}$

Rezolvare:

a) $f'(x)=(3x^2-8x+2)'=\left(3x^2\right)'-(8x)'+2'=3\cdot\left(x^2\right)'-8\cdot x'+0=3\cdot 2x-8=6x-8$;

am folosit formulele: $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $c'=0$; $x'=1$; $(x^n)'=n\cdot x^{n-1}$

b) $f'(x)=(x^2+2^x)'=\left(x^2\right)'+\left(2^x\right)'=2x+2^x\ln 2$;

am folosit formulele: $(f+g)'=f'+g'$; $(x^n)'=n\cdot x^{n-1}$; $\left(a^x\right)'=a^x\cdot\ln a$

c) $f'(x)=(e^x-5^x)'=\left(e^x\right)'-\left(5^x\right)'=e^x-5^x\ln 5$;

am folosit formulele: $(f+g)'=f'+g'$; $\left(a^x\right)'=a^x\cdot\ln a$; $\left(e^x\right)'=e^x$

d) $$f'(x)=\left(5x^4+\frac{1}{x}+\frac{1}{x^3}\right)'=\left(5x^4\right)'+\left(\frac{1}{x}\right)'+\left(\frac{1}{x^3}\right)'=5\cdot 4x^3+\left(x^{-1}\right)'+\left(x^{-3}\right)'=20x^3-1\cdot x^{-2}-3\cdot x^{-4}$$
$$=20x^3-\frac{1}{x^2}-\frac{3}{x^4};$$

am folosit formulele: $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $(x^n)'=n\cdot x^{n-1}$; $\left(x^r\right)'=r\cdot x^{r-1}$

e) $f'(x)=\left(\frac{x-4}{x}\right)'=\frac{(x-4)'\cdot x-(x-4)\cdot x'}{x^2}=\frac{1\cdot x-(x-4)\cdot 1}{x^2}=\frac{x-x+4}{x^2}=\frac{4}{x^2}$;

am folosit formulele: $\left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$; $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $c'=0$; $x'=1$.

f) $$f'(x)=\left(\frac{x-4}{x^2+2}\right)'=\frac{(x-4)'\left(x^2+2\right)-(x-4)\left(x^2+2\right)'}{\left(x^2+2\right)^2}=\frac{1\cdot\left(x^2+2\right)-(x-4)(2x)}{\left(x^2+2\right)^2}=\frac{x^2+2-2x^2+8x}{\left(x^2+2\right)^2}$$
$$=\frac{-x^2+8x+2}{\left(x^2+2\right)^2};$$

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am folosit formulele: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$; $(f+g)' = f' + g'$; $c' = 0$; $x' = 1$; $(x^n)' = n \cdot x^{n-1}$

g) $f'(x) = [(x^5 - 7x + 2)e^x]' = (x^5 - 7x + 2)' e^x + (x^5 - 7x + 2)(e^x)' = (5x^4 - 7)e^x + (x^5 - 7x + 2)e^x$
$= (x^5 + 5x^4 - 7x - 5)e^x$;

am folosit formulele:

$(f \cdot g)' = f' \cdot g + f \cdot g'$; $(f+g)' = f' + g'$; $c' = 0$; $x' = 1$; $(x^n)' = n \cdot x^{n-1}$; $(e^x)' = e^x$

h) $f'(x) = (x\ln 2 + \ln x)' = (\ln 2)x' + (\ln x)' = (\ln 2)\cdot 1 + \frac{1}{x} = \frac{1}{x} + \ln 2$;

am folosit formulele: $(f+g)' = f' + g'$; $x' = 1$; $(\ln x)' = \frac{1}{x}$; $(c \cdot f)' = c \cdot f'$

i) $f(x) = [(x+3)\ln x]' = (x+3)' \ln x + (x+3)(\ln x)' = 1 \cdot \ln x + (x+3) \cdot \frac{1}{x} = \ln x + \frac{x+3}{x}$;

am folosit formulele: $(f \cdot g)' = f' \cdot g + f \cdot g'$; $x' = 1$; $c' = 0$; $(\ln x)' = \frac{1}{x}$;

j) $f'(x) = (2\sin x + \cos x)' = (2\sin x)' + (\cos x)' = 2\cos x - \sin x$;

am folosit formulele: $(f+g)' = f' + g'$; $(c \cdot f)' = c \cdot f'$; $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$

k) $f'(x) = (2x\sin x)' = (2x)' \cdot \sin x + 2x \cdot (\sin x)' = 2\sin x + 2x \cdot (-\cos x) = 2\sin x - 2x\cos x$

am folosit formulele: $(f \cdot g)' = f' \cdot g + f \cdot g'$; $(c \cdot f)' = c \cdot f'$; $x' = 1$; $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$

l) $f'(x) = (5\arcsin x - 8\arccos x)' = (5\arcsin x)' - (8\arccos x)' = \frac{5}{\sqrt{1-x^2}} + \frac{8}{\sqrt{1-x^2}} = \frac{13}{\sqrt{1-x^2}}$;

am folosit formulele: $(f+g)' = f' + g'$; $(c \cdot f)' = c \cdot f'$; $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$; $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

m) $f'(x) = [(x^2+3)\arcsin x]' = (x^2+3)' \arcsin x + (x^2+3)(\arcsin x)' = 2x\arcsin x + \frac{x^2+3}{\sqrt{1-x^2}}$;

am folosit formulele: $(f \cdot g)' = f' \cdot g + f \cdot g'$; $(x^n)' = n \cdot x^{n-1}$; $c' = 0$; $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$$f'(x) = \left(\frac{3\sin x - 5}{\cos x + 2}\right)' = \frac{(3\sin x - 5)'(\cos x + 2) - (3\sin x - 5)(\cos x + 2)'}{(\cos x + 2)^2}$$

n) $$= \frac{3\cos x(\cos x + 2) - (3\sin x - 5)(-\sin x)}{(\cos x + 2)^2} = \frac{3\cos^2 x + 6\cos x + 3\sin^2 x - 5\sin x}{(\cos x + 2)^2};$$

$$= \frac{3(\cos^2 x + \sin^2 x) + 6\cos x - 5\sin x}{(\cos x + 2)^2} = \frac{3 + 6\cos x - 5\sin x}{(\cos x + 2)^2}$$

am folosit formulele: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$; $(f+g)' = f' + g'$; $c' = 0$; $(c \cdot f)' = c \cdot f'$;

$(\sin x)' = \cos x$; $(\cos x)' = -\sin x$;

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şi formula fundamentală a trigonometriei $\boxed{\cos^2 x+\sin^2 x=1}$

o) $f'(x)=(2arctgx+5arcctgx)'=(2arctgx)'+(5arcctgx)'=\frac{2}{1+x^2}-\frac{5}{1+x^2}=-\frac{3}{1+x^2}$

am folosit formulele: $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $(arctgx)'=\frac{1}{1+x^2}$; $(arcctgx)'=-\frac{1}{1+x^2}$

p) $f'(x)=\left(3\sqrt{x}+\sqrt[5]{x^3}\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x^2}}\right)'=3\left(\sqrt{x}\right)'+\left(\sqrt[5]{x^3}\right)'+\left(\frac{1}{\sqrt{x}}\right)'+\left(\frac{1}{\sqrt[3]{x^2}}\right)'$

$=\frac{3}{2\sqrt{x}}+\left(x^{\frac{3}{5}}\right)'+\left(x^{-\frac{1}{2}}\right)'+\left(x^{-\frac{2}{3}}\right)'=\frac{3}{2\sqrt{x}}+\frac{3}{5}x^{-\frac{2}{5}}-\frac{1}{2}x^{-\frac{3}{2}}-\frac{2}{3}x^{-\frac{5}{3}}=\frac{3}{2\sqrt{x}}+\frac{3}{5\sqrt[5]{x^2}}-\frac{1}{2\sqrt{x^3}}-\frac{2}{3\sqrt[3]{x^5}}$

am folosit formulele: $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $\sqrt[n]{x^m}=x^{\frac{m}{n}}$; $\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$; $\left(x^r\right)'=r\cdot x^{r-1}$

Exerciţii propuse

1.Să se calculeze derivatele funcţiilor, scriindu-se de fiecare dată şi formulele utilizate:

a) $f:\mathbb{R}\to\mathbb{R},\ f(x)=-8x^2+x-12$; b) $f:\mathbb{R}\to\mathbb{R},\ f(x)=3x^5+7^x$;

c) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2e^x+2^x-4^x$; d) $f:\mathbb{R}\setminus\{0\}\to\mathbb{R},\ f(x)=3x^7+\frac{1}{x^2}-\frac{1}{x^4}$;

e) $f:\mathbb{R}\setminus\{-1\}\to\mathbb{R},\ f(x)=\frac{x-3}{x+1}$; f) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{-x+3}{x^2+1}$;

g) $f:\mathbb{R}\to\mathbb{R},\ f(x)=(x^2+3x-2)e^x$; h) $f:(0;+\infty)\to\mathbb{R},\ f(x)=x\ln 7+2\ln x$;

i) $f:(0;+\infty)\to\mathbb{R},\ f(x)=(x^2+2)\ln x$; j) $f:\mathbb{R}\to\mathbb{R},\ f(x)=-3\sin x+2\cos x$;

k) $f:\mathbb{R}\to\mathbb{R},\ f(x)=(3x+4)\cos x$; l) $f:(-1;1)\to\mathbb{R},\ f(x)=4\arcsin x+3\arccos x$;

m) $f:(-1;1)\to\mathbb{R},\ f(x)=(2x-3)\arcsin x$; n) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\frac{2\cos x-3}{\sin x+2}$;

o) $f:\mathbb{R}\to\mathbb{R},\ f(x)=6arctgx-arcctgx$; p) $f:(0;+\infty)\to\mathbb{R},\ f(x)=2\sqrt{x}+\sqrt[4]{x}-\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[5]{x^3}}$

2.Construiţi alte 3 funcţii asemănătoare pentru fiecare subpunct de la ex. 1 şi calculaţi derivatele lor .