**Clasa XI – M2 – prof. Cornelia Mestecan**
**Fişă de lucru – derivate funcţii compuse**

**Exerciţii rezolvate**
Să se calculeze derivatele funcţiilor, scriindu-se de fiecare dată şi formulele utilizate:

a) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\left(3x^2-8\right)^3$; b) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2^{3x-5}$ ;

c) $f:\mathbb{R}\to\mathbb{R},\ f(x)=3e^{x^2+4}$; d) $f:\mathbb{R}\setminus\{-2;3\}\to\mathbb{R},\ f(x)=\dfrac{1}{x+2}+\dfrac{1}{(x-3)^3}$;

e) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\sin\left(2x^2+3\right)$; f) $f:(-1;+\infty)\to\mathbb{R},\ f(x)=\ln(x+1)$;

g) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\left(x^2-3x+1\right)e^{x+1}$; h) $f:(4;+\infty)\to\mathbb{R},\ f(x)=\ln\dfrac{x-4}{x^2+2}$;

i) $f:\mathbb{R}\setminus\{0\}\to\mathbb{R},\ f(x)=(x-1)\ln x^2$; j) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2\sin(2x+3)+\cos\left(3x^2\right)$;

k) $f:\mathbb{R}\to\mathbb{R},\ f(x)=x\sin 2x$; l) $f:\left(-\dfrac{1}{3};\dfrac{1}{3}\right)\to\mathbb{R},\ f(x)=\arcsin 2x+\arccos 3x$;

m) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\sin^2 x+3\cos^2 x$; n) $f:(0;+\infty)\to\mathbb{R},\ f(x)=2arctg\sqrt{x}$;

o) $f:(-2;+\infty)\to\mathbb{R},\ f(x)=\sqrt{x+2}$; p) $f:\left(-\infty;-\sqrt{2}\right)\cup\left(\sqrt{2};+\infty\right)\to\mathbb{R},\ f(x)=\sqrt{\dfrac{x^2-2}{x^2}}$

Rezolvare:

a) $f'(x)=\left[\left(3x^2-8\right)^3\right]'=3\cdot\left(3x^2-8\right)^2\cdot\left(3x^2-8\right)'=18x\left(3x^2-8\right)^2$

am folosit formulele: $(f\circ u)'=(f(u))'=f'(u)\cdot u'$; $\left[u^3(x)\right]'=3u^2(x)\cdot u'(x)$;
$(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $(x^n)'=n\cdot x^{n-1}$; $c'=0$

b) $f'(x)=\left(2^{3x-5}\right)'=2^{3x-5}\cdot\ln 2\cdot(3x-5)'=2^{3x-5}\cdot 3\ln 2$

am folosit formulele: $(f\circ u)'=(f(u))'=f'(u)\cdot u'$; $\left(a^{u(x)}\right)'=a^{u(x)}\cdot\ln a\cdot u'(x)$; $(f+g)'=f'+g'$;
$(c\cdot f)'=c\cdot f'$; $c'=0$

c) $f'(x)=\left(3e^{x^2+4}\right)'=3e^{x^2+4}\cdot\left(x^2+4\right)'=6xe^{x^2+4}$;

am folosit formulele: $(f\circ u)'=(f(u))'=f'(u)\cdot u'$; $\left(e^{u(x)}\right)'=e^{u(x)}\cdot u'(x)$;
$(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $(x^n)'=n\cdot x^{n-1}$; $c'=0$

d)

$$f'(x)=\left[\frac{1}{x+2}+\frac{1}{(x-3)^3}\right]'=\left(\frac{1}{x+2}\right)'+\left(\frac{1}{(x-3)^3}\right)'=-\frac{(x+2)'}{(x+2)^2}-\frac{3(x-3)'}{(x-3)^4}=-\frac{1}{(x+2)^2}-\frac{3}{(x-3)^4};$$

am folosit formulele: $(f\circ u)'=(f(u))'=f'(u)\cdot u'$; $\left(u^r(x)\right)'=r\cdot u^{r-1}(x)\cdot u'(x)$; $(f+g)'=f'+g'$;
$c'=0$; $x'=1$

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$$\left(\frac{1}{u(x)}\right)' = \left(u^{-1}(x)\right)' = -u^{-2}(x)\cdot u'(x) = -\frac{u'(x)}{u^2(x)};\ \left(\frac{1}{u^3(x)}\right)' = \left(u^{-3}(x)\right)' = -3u^{-4}(x)\cdot u'(x) = -\frac{3u'(x)}{u^4(x)}$$

e) $f'(x) = \left[\sin\left(2x^2+3\right)\right]' = \left(\cos\left(2x^2+3\right)\right)\cdot\left(2x^2+3\right)' = 4x\cos\left(2x^2+3\right);$

am folosit formulele: $(f\circ u)' = (f(u))' = f'(u)\cdot u'$; $\left[\sin u(x)\right]' = \left(\cos u(x)\right)\cdot u'(x)$;
$(f+g)' = f'+g'$; $(c\cdot f)' = c\cdot f'$; $(x^n)' = n\cdot x^{n-1}$; $c'=0$

f) $f'(x) = \left[\ln(x+1)\right]' = \dfrac{(x+1)'}{x+1} = \dfrac{1}{x+1};$

am folosit formulele: $(f\circ u)' = (f(u))' = f'(u)\cdot u'$; $\left(\ln u(x)\right)' = \dfrac{1}{u(x)}\cdot u'(x) = \dfrac{u'(x)}{u(x)}$;

$(f+g)' = f'+g'$; $c'=0$; $x'=1$

g)
$$f'(x) = \left[\left(x^2-3x+1\right)e^{x+1}\right]' = \left(x^2-3x+1\right)' e^{x+1} + \left(x^2-3x+1\right)\left(e^{x+1}\right)'$$
$$= (2x-3)e^{x+1} + \left(x^2-3x+1\right)\left(e^{x+1}\right)(x+1)' = (2x-3)e^{x+1} + \left(x^2-3x+1\right)\left(e^{x+1}\right)\cdot 1;$$
$$= e^{x+1}\left(x^2-x-2\right)$$

am folosit formulele: $(f\cdot g)' = f'\cdot g + f\cdot g'$; $(f\circ u)' = (f(u))' = f'(u)\cdot u'$; $\left(e^{u(x)}\right)' = e^{u(x)}\cdot u'(x)$;
$(f+g)' = f'+g'$; $(c\cdot f)' = c\cdot f'$; $(x^n)' = n\cdot x^{n-1}$; $c'=0$; $x'=1$

h)
$$f'(x) = \left(\ln\frac{x-4}{x^2+2}\right)' = \frac{1}{\frac{x-4}{x^2+2}}\cdot\left(\frac{x-4}{x^2+2}\right)' = \frac{x^2+2}{x-4}\cdot\frac{(x-4)'\left(x^2+2\right)-(x-4)\left(x^2+2\right)'}{\left(x^2+2\right)^2};$$
$$= \frac{1\cdot\left(x^2+2\right)-(x-4)\cdot 2x}{(x-4)\left(x^2+2\right)} = \frac{x^2+2-2x^2+8x}{(x-4)\left(x^2+2\right)} = \frac{-x^2+8x+2}{(x-4)\left(x^2+2\right)}$$

am folosit formulele: $(f\circ u)' = (f(u))' = f'(u)\cdot u'$; $\left(\ln u(x)\right)' = \dfrac{1}{u(x)}\cdot u'(x)$

$\left(\dfrac{f}{g}\right)' = \dfrac{f'\cdot g - f\cdot g'}{g^2}$; $(f+g)' = f'+g'$; $(c\cdot f)' = c\cdot f'$; $(x^n)' = n\cdot x^{n-1}$; $c'=0$; $x'=1$

i)
$$f'(x) = \left((x-1)\ln x^2\right)' = (x-1)'\cdot\ln x^2 + (x-1)\cdot\left(\ln x^2\right)' = 1\cdot\ln x^2 + (x-1)\cdot\frac{\left(x^2\right)'}{x^2}$$
$$= \ln x^2 + (x-1)\cdot\frac{2x}{x^2} = \ln x^2 + \frac{2(x-1)}{x}$$

am folosit formulele: $(f\circ u)' = (f(u))' = f'(u)\cdot u'$; $\left(\ln u(x)\right)' = \dfrac{1}{u(x)}\cdot u'(x) = \dfrac{u'(x)}{u(x)}$

$(f\cdot g)' = f'\cdot g + f\cdot g'$; $(f+g)' = f'+g'$; $(x^n)' = n\cdot x^{n-1}$; $c'=0$; $x'=1$

j) $$f'(x)=\left[2\sin(2x+3)+\cos\left(3x^2\right)\right]'=\left(2\sin(2x+3)\right)'+\left(\cos\left(3x^2\right)\right)'$$
$$=2\cos(2x+3)\cdot(2x+3)'-\sin\left(3x^2\right)\cdot\left(3x^2\right)'=4\cos(2x+3)-6x\sin\left(3x^2\right);$$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$;

$(x^n)'=n\cdot x^{n-1}$; $c'=0$; $x'=1$; $\left(\sin u(x)\right)'=\left(\cos u(x)\right)\cdot u'(x)$; $\left(\cos u(x)\right)'=\left(-\sin u(x)\right)\cdot u'(x)$

k) $f'(x)=\left(x\sin 2x\right)'=x'\sin 2x+x\left(\sin 2x\right)'=\sin 2x+x\left(\cos 2x\right)\cdot(2x)'=\sin 2x+2x\left(\cos 2x\right)$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $\left(\sin u(x)\right)'=\left(\cos u(x)\right)\cdot u'(x)$;

$(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $x'=1$

l) $$f'(x)=\left(\arcsin 2x+\arccos 3x\right)'=\left(\arcsin 2x\right)'+\left(\arccos 3x\right)'=\frac{(2x)'}{\sqrt{1-(2x)^2}}-\frac{(3x)'}{\sqrt{1-(3x)^2}};$$
$$=\frac{2}{\sqrt{1-4x^2}}-\frac{3}{\sqrt{1-9x^2}}$$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $\left(\arcsin u(x)\right)'=\dfrac{u'(x)}{\sqrt{1-u^2(x)}}$;

$\left(\arccos u(x)\right)'=-\dfrac{u'(x)}{\sqrt{1-u^2(x)}}$; $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$; $x'=1$

m) $$f'(x)=\left(\sin^2 x+3\cos^2 x\right)'=\left(\sin^2 x\right)'+\left(3\cos^2 x\right)'=2\sin x\cdot(\sin x)'+6\cos x\cdot(\cos x)'$$
$$=2\sin x\cos x-6\cos x\sin x=-4\sin x\cos x$$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $(f+g)'=f'+g'$; $(c\cdot f)'=c\cdot f'$;

$\left(\sin u(x)\right)'=\left(\cos u(x)\right)\cdot u'(x)$; $\left(\cos u(x)\right)'=-\left(\sin u(x)\right)\cdot u'(x)$

n) $$f'(x)=\left(2arctg\sqrt{x}\right)'=2\frac{\left(\sqrt{x}\right)'}{1+\left(\sqrt{x}\right)^2}=\frac{2\frac{1}{2\sqrt{x}}}{1+x}=\frac{1}{\sqrt{x}}\cdot\frac{1}{1+x}=\frac{1}{\sqrt{x}(1+x)};$$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $(c\cdot f)'=c\cdot f'$;

$\left(arctgu(x)\right)'=\dfrac{1}{1+u^2(x)}\cdot u'(x)=\dfrac{u'(x)}{1+u^2(x)}$; $\left(\sqrt{x}\right)'=\dfrac{1}{2\sqrt{x}}$

o) $$f'(x)=\left(\sqrt{x+2}\right)'=\frac{(x+2)'}{2\sqrt{x+2}}=\frac{1}{2\sqrt{x+2}};$$

am folosit formulele: $(f\circ u)'=\left(f(u)\right)'=f'(u)\cdot u'$; $(f+g)'=f'+g'$; $c'=0$; $x'=1$;

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$\left(\sqrt{u(x)}\right)' = \dfrac{1}{2\sqrt{u(x)}} \cdot u'(x) = \dfrac{u'(x)}{2\sqrt{u(x)}}$

p)
$$f'(x) = \left(\sqrt{\frac{x^2-2}{x^2}}\right)' = \frac{1}{2\sqrt{\frac{x^2-2}{x^2}}} \cdot \left(\frac{x^2-2}{x^2}\right)' = \frac{1}{2}\sqrt{\frac{x^2}{x^2-2}} \cdot \frac{\left(x^2-2\right)' \cdot x^2 - \left(x^2-2\right) \cdot \left(x^2\right)'}{\left(x^2\right)^2}$$

$$= \frac{1}{2}\sqrt{\frac{x^2}{x^2-2}} \cdot \frac{2x \cdot x^2 - \left(x^2-2\right) \cdot 2x}{x^4} = \frac{1}{2}\sqrt{\frac{x^2}{x^2-2}} \cdot \frac{2x^3 - 2x^3 + 4x}{x^4} = \frac{1}{2}\sqrt{\frac{x^2}{x^2-2}} \cdot \frac{4x}{x^4} = \frac{1}{x^3}\sqrt{\frac{x^2}{x^2-2}}$$

am folosit formulele: $(f \circ u)' = (f(u))' = f'(u) \cdot u'$; $\left(\sqrt{u(x)}\right)' = \dfrac{1}{2\sqrt{u(x)}} \cdot u'(x) = \dfrac{u'(x)}{2\sqrt{u(x)}}$ **;**

$\left(\dfrac{f}{g}\right)' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$ **;** $(f+g)' = f' + g'$ **;** $c' = 0$ **;** $(x^n)' = n \cdot x^{n-1}$ **;**

**Exerciţii propuse**

1.Să se calculeze derivatele funcţiilor, scriindu-se de fiecare dată şi formulele utilizate:

a) $f:\mathbb{R}\to\mathbb{R}, f(x) = \left(-x^2+2x-4\right)^5$; b) $f:\mathbb{R}\to\mathbb{R}, f(x) = 2^{-2x+1}$ ;

c) $f:\mathbb{R}\to\mathbb{R}, f(x) = 5e^{3x^2-7}$; d) $f:\mathbb{R}\setminus\{-3;4\}\to\mathbb{R}, f(x) = \dfrac{1}{x-4} + \dfrac{1}{(x+3)^2}$;

e) $f:\mathbb{R}\to\mathbb{R}, f(x) = \sin\left(5x^2+1\right)$; f) $f:(3;+\infty)\to\mathbb{R}, f(x) = \ln(x-3)$;

g) $f:\mathbb{R}\to\mathbb{R}, f(x) = \left(x^2+x+3\right)e^{-x+1}$; h) $f:(-3;+\infty)\to\mathbb{R}, f(x) = \ln\dfrac{x+3}{x^2+5}$;

i) $f:\mathbb{R}\to\mathbb{R}, f(x) = (x+2)\ln\left(x^2+4\right)$; j) $f:\mathbb{R}\to\mathbb{R}, f(x) = 2\sin(-2x+1) + \cos\left(x^2+1\right)$;

k) $f:\mathbb{R}\to\mathbb{R}, f(x) = (x-3)\sin 5x$; l) $f:\left(-\dfrac{1}{3};\dfrac{1}{3}\right)\to\mathbb{R}, f(x) = \arcsin 3x + \arccos 2x$;

m) $f:\mathbb{R}\to\mathbb{R}, f(x) = 5\sin^2 x + 2\cos^2 x$; n) $f:(-4;+\infty)\to\mathbb{R}, f(x) = 3arctg\sqrt{x+4}$;

o) $f:(7;+\infty)\to\mathbb{R}, f(x) = \sqrt{x-7}$; p) $f:(-\infty;-2)\cup(2;+\infty)\to\mathbb{R}, f(x) = \sqrt{\dfrac{x^2-4}{x^2+1}}$

2.Construiţi alte 3 funcţii asemănătoare pentru fiecare subpunct de la ex. 1 şi calculaţi derivatele lor .