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Calculul primitivelor

Reguli de calcul cu primitive

$I \subset R, f, g: I \to R$, funcţii care admit primitive pe $I, \alpha, \beta \in R$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int [\alpha f(x) + \beta g(x)]dx = \alpha \int f(x)dx + \beta \int g(x)\, dx$$

Exerciţiul 1

$$\int (x^3 + 2x - 1)dx = ?$$

$$\int x^3 dx + \int 2x dx - \int 1 dx = \frac{x^4}{4} + 2\int x dx - x = \frac{x^4}{4} + 2\frac{x^2}{2} - x + C$$

Formule pe care le folosim:

$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ $\int 1 dx = x + C$

Exerciţiul 2

$$\int (3x^7 - 4x^5 + 121)dx = ?$$

$$3\int x^7 dx - 4\int x^5 dx + 121\int 1dx$$

$$3\frac{x^8}{8} - 4\frac{x^6}{6} + 121x + C$$

Exerciţiul 3

$$\int \left(3x^2 - \frac{1}{x^5} + \frac{2}{x^2} - \frac{1}{x}\right) dx = ?$$

$\boldsymbol{\frac{1}{x^n} = x^{-n}}$ $\frac{1}{x^5} = x^{-5}$ $\frac{1}{x^2} = x^{-2}$

$$3\int x^2dx-\int x^{-5}dx+2\int x^{-2}dx-\int \frac{1}{x}dx$$

$$\int \frac{\mathbf{1}}{\boldsymbol{x}}\boldsymbol{dx}=\mathbf{ln}|\boldsymbol{x}|+\boldsymbol{C}$$

$$3\frac{x^3}{3}-\frac{x^{-5+1}}{-5+1}+2\frac{x^{-2+1}}{-2+1}-\ln|x|+C$$

$$x^3-\frac{x^{-4}}{-4}+2\frac{x^{-1}}{-1}-\ln|x|+C$$

$$x^3+\frac{x^{-4}}{4}-2x^{-1}-\ln|x|+C$$

Exerciţiul 4

$$\int\left(-7x^3-\frac{1}{x^3}+\frac{2}{x^4}+\frac{1}{x}\right)dx=?$$

$$-\mathbf{7}\int x^3dx-\int x^{-3}dx+2\int x^{-4}dx+\ln|x|$$

$$\int \boldsymbol{x^n}\,\boldsymbol{dx}=\frac{\boldsymbol{x^{n+1}}}{\boldsymbol{n+1}}+\boldsymbol{C}$$

$$\int \frac{\mathbf{1}}{\boldsymbol{x}}\boldsymbol{dx}=\mathbf{ln}|\boldsymbol{x}|+\boldsymbol{C}$$

$$-7\frac{x^4}{4}-\frac{x^{-2}}{-2}+2\frac{x^{-3}}{-3}+\ln|x|+C$$

$$-\frac{7}{4}x^4+\frac{1}{2}x^{-2}-\frac{2}{3}x^{-3}+\ln|x|+C$$

Exerciţiul 5

$$\int\left(3e^x-2^x-\frac{3}{x^3}-\frac{2}{x}\right)dx=?$$

$$\int \boldsymbol{a^x}\boldsymbol{dx}=\frac{\boldsymbol{a^x}}{\mathbf{ln}\,\boldsymbol{a}}+\boldsymbol{C} \qquad \mathbf{ln}\,\boldsymbol{e}=\mathbf{1},\quad \mathbf{ln}\,\mathbf{1}=\mathbf{0}$$

$$\int \boldsymbol{e^x dx = e^x + C}$$

$$3\int e^x dx - \int 2^x dx - 3\int \frac{1}{x^3} dx - 2\int \frac{1}{x} dx$$

$$3e^x - \frac{2^x}{\ln 2} - 3\int x^{-3} dx - 2\ln|x|$$

$$3e^x - \frac{2^x}{\ln 2} - 3\frac{x^{-2}}{-2} - 2\ln|x| + C$$

Formule:

$$\boldsymbol{\frac{1}{x^n} = x^{-n}}$$

$$\int \boldsymbol{\frac{1}{x} dx = \ln|x| + C}$$

Exercițiul 6

$$\int \left(2\sqrt{x} - 7^x - \frac{3}{\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x^2}}\right) dx = ?$$

Formulele pe care le folosim:

$$\boldsymbol{\sqrt[n]{x^m} = x^{\frac{m}{n}}} \qquad \int \boldsymbol{x^a\, dx = \frac{x^{a+1}}{a+1} + C} \qquad \int \boldsymbol{1 dx = x + C}$$

$$\boldsymbol{\frac{1}{x^n} = x^{-n}} \ \ ; \int \boldsymbol{a^x dx = \frac{a^x}{\ln a} + C}$$

Rezolvare:

$$\int \left(2\sqrt{x} - 7^x - \frac{3}{\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x^2}}\right) dx = 2\int x^{\frac{1}{2}} dx - \frac{7^x}{\ln 7} - 3\int x^{-\frac{1}{3}} dx + 2\int x^{-\frac{2}{3}} dx$$

$$= 2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{7^x}{\ln 7} - 3\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + 2\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C$$

$$= 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{7^x}{\ln 7} - 3\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 2\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C$$

$$= 2\cdot\frac{2}{3}x^{\frac{3}{2}} - \frac{7^x}{\ln 7} - 3\cdot\frac{3}{2}x^{\frac{2}{3}} + 2\cdot\frac{3}{1}x^{\frac{1}{3}} + C$$

$$= \frac{4}{3}x^{\frac{3}{2}} - \frac{7^x}{\ln 7} - \frac{9}{2}x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + C$$

Exerciţiul 7

$$\int (3\sin x - 4\cos x)dx = ?$$

Formule folosite:

$$\int \sin x\, dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C$$

Rezolvare:

$$\int (3\sin x - 4\cos x)dx$$

$$= 3\int \sin x\, dx - 4\int \cos x\, dx = 3(-\cos x) - 4\sin x + C$$

Exerciţiul 8 în clasă

$$\int (5\cos x - \sin x)dx = ?$$

Exerciţiul 9

$$\int \left(2x^4 - 3 \cdot 4^x - \frac{1}{x^2+1} + \frac{2}{x^2+3}\right) dx = ?$$

Formule folosite:

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C; \quad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} arctg \frac{x}{a} + C$$

Rezolvare:

$$\int \left(2x^4 - 3 \cdot 4^x - \frac{1}{x^2+1} + \frac{2}{x^2+3}\right) dx$$
$$= 2\int x^4 dx$$
$$- 3\int 4^x dx - \int \frac{1}{x^2+1} dx + 2\int \frac{1}{x^2+3} dx$$

$$= 2\frac{x^5}{5} - 3\frac{4^x}{\ln 4} - \underbrace{\frac{1}{1} arctg \frac{x}{1}}_{a^2=1 \Rightarrow a=1} + 2\underbrace{\frac{1}{\sqrt{3}} arctg \frac{x}{\sqrt{3}}}_{a^2=3 \Rightarrow a=\sqrt{3}} + C$$

Exerciţiul 10 în clasă

$$\int \left(\frac{7}{x^2+2} + \frac{2}{x^2+5}\right) dx = ?$$

Exerciţiul 11

$$\int \left(\frac{1}{x^2+3} + \frac{2}{x^2-3}\right) dx = ?$$

Formule folosite

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a} arctg\frac{x}{a} + C$$

Rezolvare:

$$\int \left(\frac{1}{x^2+3}+\frac{2}{x^2-5}\right)dx = \int \frac{1}{x^2+3}dx + 2\int \frac{1}{x^2-3}dx = \frac{1}{\sqrt{3}} arctg\frac{x}{\sqrt{3}} +$$

$$2\cdot\frac{1}{2\sqrt{5}}\ln\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + C$$

Pentru prima integrală $a^2 = 3 \Rightarrow a = \sqrt{3}$ şi pentru a doua $a^2 = 5 \Rightarrow a = \sqrt{5}$

Exerciţiul 12 în clasă

$$\int \left(\frac{4}{x^2-11}+\frac{5}{x^2-13}\right)dx = ?$$

Exerciţiul 13

$$\int \left(\frac{15}{\sqrt{x^2-6}}+\frac{2}{\sqrt{x^2+5}}\right)dx = ?$$

Formule folosite:

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

Rezolvare:

$$\int \left(\frac{15}{\sqrt{x^2-6}}+\frac{2}{\sqrt{x^2+5}}\right)dx = 15\int \frac{1}{\sqrt{x^2-6}}dx + 2\int \frac{1}{\sqrt{x^2+5}}dx$$
$$= 15\ln\left|x+\sqrt{x^2-6}\right| + 2\ln\left(x+\sqrt{x^2+5}\right) + C$$

Exerciţiul 14 în clasă

$$\int \left(\frac{2}{\sqrt{x^2-1}}+\frac{17}{\sqrt{x^2+7}}\right)dx = ?$$

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Exerciţiul 15

$$\int \left(\frac{6}{\sqrt{x^2+15}}+\frac{2}{\sqrt{7-x^2}}\right)dx = ?$$

Formule folosite:

$$\int \frac{\mathbf{1}}{\sqrt{\boldsymbol{x^2+a^2}}}\boldsymbol{dx} = \mathbf{ln}\left(x+\sqrt{\boldsymbol{x^2+a^2}}\right)+\boldsymbol{C}$$

$$\int \frac{\mathbf{1}}{\sqrt{\boldsymbol{a^2-x^2}}}\boldsymbol{dx} = \boldsymbol{arcsin}\frac{\boldsymbol{x}}{\boldsymbol{a}}+\boldsymbol{C}$$

Rezolvare:

$$\int \left(\frac{6}{\sqrt{x^2+15}}+\frac{2}{\sqrt{7-x^2}}\right)dx = 6\int \frac{1}{\sqrt{x^2+15}}dx + 2\int \frac{1}{\sqrt{7-x^2}}dx$$

$$= 6\ln\left(x+\sqrt{x^2+15}\right)+2arcsin\frac{x}{\sqrt{7}}+C$$

La a doua integrală am avut $a^2 = 7 \Rightarrow a = \sqrt{7}$

Exerciţiul 16 în clasă

$$\int \left(\frac{7}{\sqrt{x^2+8}}+\frac{2}{\sqrt{11-x^2}}\right)dx = ?$$

Temă

Calculaţi:

1. $\int\left(x^7-2x^3+3x-5\right)dx$; 2. $\int\left(3x^7-7^x\right)dx$; 3. $\int\left(2\cos x+5\sin x\right)dx$;

4. $\int\left(\frac{2}{x}-\frac{1}{x^7}\right)dx$; 5. $\int\left(\sqrt[3]{x}-\frac{1}{\sqrt{x}}\right)dx$; 6. $\int\left(\frac{1}{x^2+7}-\frac{1}{x^2-7}\right)dx$

7. $\int\left(\frac{2}{\sqrt{x^2-11}}+\frac{1}{\sqrt{x^2+11}}\right)$